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ERRATUM

The Publishers apologise for errors which occurred in the original printing of this article through misunderstanding of the author's proof corrections. In view of the importance which the author attaches to this article it is republished in full.

AN ELASTIC–PLASTIC ANALYSIS OF A BAR UNDER REPEATED AXIAL LOADING

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Abstract—An elastic–perfectly plastic analysis of a bar under repeated axial loading is herein presented. The bar is taken as a one-dimensional continuum with both ends simply supported. The plastic interaction is considered for the combined action of bending and axial deformation, based on a piecewise-linear yield condition. The solution is derived in a closed form, and can describe the hysteretic behavior of a bar, such as a structural brace or a truss member, under any given history of tension and/or compression or of corresponding displacements. An example is illustrated, and some implications of the analytical results are discussed. The paper is concluded with critical comments on the basic assumptions adopted in the course of the analysis.

1. INTRODUCTION

THE role of bracing in the structural behavior of a frame cannot be overemphasized. A great deal of contribution is normally anticipated from braces to the strength and rigidity of a braced frame. The structural functions of braces are represented by their capabilities to sustain axial loads. Therefore the knowledge of the axial load–displacement relationships is essential in finding the structural performance of a braced frame. This is also the case with a trussed structure. The overall behavior of a truss cannot be revealed without clarifying the axial load–displacement characteristics of each constituent member. Such a knowledge is prerequisite to the determination of the load-carrying capacity of a redundant structure.

When a structure is subjected to a repeated load as in an earthquake, a bracing or a truss member is likewise subjected to tension and/or compression repeatedly. An initially straight bar under compression may be forced into a plastically deformed configuration due to instability effects, with its axial-load carrying capacity much decreased. A subsequent tension may decrease the deflection and the bar will recover some strength and rigidity. This bar may again be subjected to compression before recovering its full strength. Repetition of this kind of loading complicates the hysteretic behavior of the bar.

There are some experimental data on the hysteretic behavior of steel bars under alternately repeated axial loading, their axial load–displacement relation indicating the aforementioned characteristics [1–3]. Theoretical investigations, however, seem to be scarce. Fujimoto *et al.* [4], Wakabayashi *et al.* [5] and Igarashi *et al.* [6] analyzed a braced portal frame under alternately repeated loading on the basis of a modified simple plastic theory. The effects of axial forces were considered in the yield condition for the braces, but it was not taken into account that at a yield hinge plastic action takes place in the axial deformation as well as in bending [7, 8]. In fact, plastic axial deformation at a yield hinge plays an important role in the load–deformation characteristics of a bar when subjected to a large axial force as in a brace or in a truss member.* Reference should be made in this connection to the work of Matsui *et al.* [11]. The results of their numerical analysis on the post-buckling behavior of a compressed steel brace indicate, in effect, that the axial displacement cannot be evaluated properly by the plastic hinge method which neglects the plastic axial deformation at a yield hinge.

In this paper, the hysteretic behavior of an elastic–plastic bar is to be studied theoretically for repeated action of axial loading, following the general principles of perfect plasticity. A few implications of the analytical results are also to be discussed.

2. BASIC RELATIONS AND ASSUMPTIONS

Suppose an initially straight bar with uniform cross-section is subjected to an axial load. The load is repeatedly applied and varied quasi-statically. The hysteretic behavior of the bar in equilibrium is to be analyzed. The relationship between the load N and the axial displacement Δ is of primary interest. The effective length is taken from the bar so that the bar of length l can be considered as being simply supported at its ends. It is supposed that the cross-section has an axis of symmetry, and that the bar deflects only in the plane of symmetry. N is taken positive when tensile, and Δ, defined as the relative displacement between the ends, is taken positive when the distance increases.

For mathematical simplicity, analysis is to be based on the following assumptions:

1. The bar can be considered as a one-dimensional continuum.

2. It has an elastic–perfectly plastic property, under the combined action of an axial force N and a bending moment M, shear effects being negligible. The yield condition is idealized as the one for an ideal I-section with indefinitely thin web and flanges,

$$\left|\frac{N}{N_0}\right| + \left|\frac{M}{M_0}\right| = 1, \tag{1}$$

where N_0 is the limit load in pure tension and M_0 the limit moment in pure bending.

3. Although change in geometry is taken into account, the deflections are sufficiently small so that the square of the slope can be neglected in comparison with unity. Change in the length of the bar is also negligibly small when compared with the original length.

4. When the load is compressive, the bar cannot carry a load greater in magnitude than the Euler's elastic buckling load $N_E = \pi^2 EI/l^2$, where E is the Young's modulus and I is the moment of inertia of the cross-section. In case $N_E < N_0$, the bar deflects transversely in an elastic and quasi-static manner sustaining the buckling load until the yield limit, equation (1), is reached at the critical section in the deflected configuration.

5. When subjected to the compression of magnitude N_0, a straight-shaped bar cannot remain straight, but deflects transversely. This assumption excludes an extremely stubby bar, which can contract plastically without appreciable deflection under the axial load N_0.

6. The bar is composed of ductile material, and no local instability takes place.

Let the dimensionless axial load n and displacement δ be defined by

$$n \equiv \frac{N}{N_0}, \qquad \delta \equiv \frac{EA\Delta}{N_0 l}, \tag{2}$$

* Based on the author's early notes [9], which constitute a basis for the present paper, Shibata took this plastic interaction effect into consideration in his analysis of braced frames, neglecting elastic flexural deformation of braces [10], and found a reasonable agreement with experimental results from a previous test.

where A is the cross-sectional area. δ is equal to the ratio of Δ to the original yield point displacement $N_0 l/(EA)$ under pure tension. On the basis of above assumptions, δ is made up of four components,

$$\delta = \delta^e + \delta^g + \delta^p + \delta^t, \tag{3}$$

where δ^e is due to uniform elastic axial deformation, δ^g to change in geometry caused by lateral deflection, δ^p to plastic axial deformation at a yield hinge, and δ^t to plastic elongation distributed along the bar axis.

The first component δ^e is related to the axial force by the elastic linear law

$$\delta^e = n. \tag{4}$$

Deflection $y(x)$ in the transverse direction reduces the relative displacement Δ by the amount

$$\int_0^l \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx,$$

in conformity with assumption 3, where x is the co-ordinate taken along the original bar axis from an end. With notation

$$\xi \equiv \frac{2x}{l}, \qquad \eta \equiv \frac{N_0 y}{M_0}; \qquad \alpha \equiv \frac{A}{I}\left(\frac{M_0}{N_0}\right)^2, \qquad n_E \equiv \frac{N_E}{N_0}, \tag{5}$$

and with symmetry consideration in Fig. 1, the second component δ^g in equation (3) is therefore evaluated from

$$\delta^g = -\frac{2\alpha n_E}{\pi^2}\int_0^1 (\eta')^2 \, d\xi \tag{6}$$

where the prime indicates differentiation with respect to ξ. The parameter α is a number depending only on the shape of the cross-section, and is equal to the square of the ratio of plastic section modulus to the product of cross-sectional area and radius of gyration. For an ideal I-section, $\alpha = 1$, and for a rectangular cross-section, $\alpha = 3/4$. Thus, α takes a value close to unity for most structural steel sections.

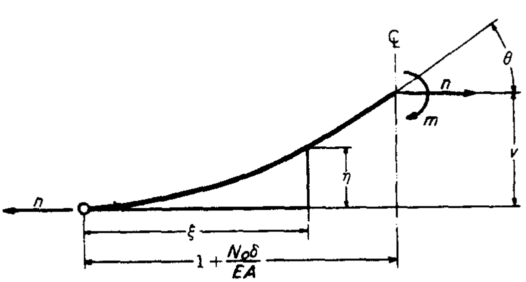

FIG. 1. Deflected half bar and dimensionless quantities.

The dimensionless deflection $\eta(\xi)$ which satisfies both the equilibrium differential equation

$$\eta'' - \nu^2 \eta = 0 \tag{7}$$

and the boundary condition at the bar end $\eta(0) = 0$ must be of the form

$$\eta = a \sinh(\nu\xi), \tag{8}$$

where

$$\nu \equiv \sqrt{\frac{Nl^2}{4EI}} = \frac{\pi}{2}\sqrt{\frac{n}{n_E}}. \tag{9}$$

The constant of integration "a" appearing in equation (8) is determined from the other boundary condition at the bar center, i.e., either

$$\eta(1) = v \quad \text{or} \quad \eta'(1-0) = \theta, \tag{10a, b}$$

according to whether plastic action takes place there or not, respectively. It is noted that v and θ have necessarily the same sign in our problem, and that the sense of η-axis is so taken that v and θ are non-negative.

Plastic action can take place at the bar center when the yield condition, equation (1), is satisfied.* Let m denote the ratio of the bending moment M at the center to M_0, its positive direction being shown in Fig. 1. The equilibrium relation for the half bar

$$m + nv = 0, \tag{11}$$

along with equation (1) give the relation

$$n = \pm\frac{1}{1+v}, \tag{12}$$

if it is noted from equation (11) that m and n have opposite signs. It follows from equations (6), (8) and (10a) that

$$\delta^g = -\alpha n_E\left(\frac{v}{\pi}\frac{\nu}{\sinh \nu}\right)^2\left[\frac{\sinh(2\nu)}{2\nu}+1\right]. \tag{13}$$

Combination of equations (9), (12) and (13) expresses δ^g in terms of n. The plastic action causes discontinuity in slope at the center. The slope angle θ at just next to the center of the dimensionless deflection curve is found from equations (8) and (10) to be

$$\theta = v\frac{\nu}{\tanh \nu}. \tag{14}$$

When the state of stress is such that $|m|+|n| < 1$, the bar behaves in an elastic manner, and θ remains constant, whose value is determined by the preceding plastic action through equation (14). In this case δ^g is given from equations (6), (8) and (10b) to be

$$\delta^g = -\alpha n_E\left(\frac{\theta}{\pi \cosh \nu}\right)^2\left[\frac{\sinh(2\nu)}{2\nu}+1\right]. \tag{15}$$

* Although the present formulation expresses δ explicitly in terms of n, owing to the irreversibility of a plastic process the uniqueness of the solution is not assured by the specification of the history of n; the history of δ determines the history of n uniquely.

The central dimensionless deflection v varies with the axial force, and is found by recourse to equations (8) and (10) to be

$$v = \theta \frac{\tanh \nu}{\nu}. \qquad (16)$$

When the axial force is compressive, n is negative, and ν and "a" are imaginary. In order to express δ^g in terms of real functions, ν simply has to be replaced by its modulus $|\nu|$, and the hyperbolic functions by the corresponding trigonometric functions in equations (13) and (15). A similar manipulation in equations (14) and (16) gives θ and v, respectively, in terms of real functions.

The third component δ^p in equation (3) varies in value when plastic action takes place at the central yield hinge. Due to the presence of axial force, plastic deformation occurs there in axial direction as well as in bending [7, 8]. Their relation is provided by the flow rule associated with the yield condition, equation (1). In geometric terms, the flow rule states that when the state of stress is represented by a point on the yield polygon, the plastic flow vector is in the direction of the outward normal to the yield polygon at the corresponding stress point [12]. By representing the yield polygon in the dimensionless stress plane (m, n), and by recalling that m and n have opposite signs [see Fig. 2(d)], it is found that, except for corners $m = \pm 1$ and $n = \pm 1$, two components of the plastic flow vector, viz., M_0 times the rate of change of the slope discontinuity and N_0 times the rate of change of the plastic elongation, are equal and opposite. With dots denoting the rates, it follows that

$$\dot{\delta}^p = -\left(\frac{2}{\pi}\right)^2 \alpha n_E \dot{\theta}. \qquad (17)$$

It should be noted that the cases $m = \pm 1$ are not encountered in our problem, and that consideration of plastic action for $n = -1$ is not required due to assumption 5. If the conditions $n = 1$ and $\dot{n} = 0$ are simultaneously satisfied, equation (14) gives in conjunction with equations (9) and (12) that $\dot{\theta} = 0$, and equation (17) leads to $\dot{\delta}^p = 0$. The plastic deformation for this case* is accounted for in the component δ^t. Equation (17) is still valid when the bar behaves elastically, because then θ remains constant and equation (17) leads to $\dot{\delta}^p = 0$. Consequently, equation (17) holds throughout the history, and integration with the initial condition $\delta^p = 0$ for $\theta = 0$ gives the relation

$$\delta^p = -\left(\frac{2}{\pi}\right)^2 \alpha n_E \theta. \qquad (18)$$

The last term δ^t in the right-hand side of equation (3) can take any non-negative value. The only restrictions are that $\dot{\delta}^t \geq 0$ during the simultaneous satisfaction of $n = 1$ and $\dot{n} = 0$, which can be attained solely in a straight configuration, and that $\dot{\delta}^t = 0$ otherwise. Evidently this is allowed by the generalized flow rule stipulated for the corner of the yield polygon that the plastic flow vector lies between the outward normals to the adjacent line segments [13].

Worth noting is that $\delta^g \leq 0$, $\delta^p \leq 0$ and $\delta^t \geq 0$, whereas δ^e can have either sign.

* When $n = 1$ and $\dot{n} < 0$, plastic action does not occur, and hence $\dot{\theta} = 0$. Assumption of perfect plasticity does not allow the case of $n = 1$, $\dot{n} > 0$. With these in mind it is found that $\dot{\theta} = \dot{\delta}^p = 0$ whenever $n = 1$.

3. EXAMPLE

Let us consider an example, in which the bar is first subjected to elongation from the straight virgin state until the yield limit $n = 1$ is reached. The behavior in this process is obvious, the n–δ relation being represented by the trace ⓪ → ① → ② in Fig. 2(a). It is instructive to illustrate the corresponding histories of deformation and of the state of stress at the critical section; in (b), (c) and (d) of Fig. 2, v, θ and m are taken, respectively, for the abscissae as against the common ordinate of n.

After undergoing the deformation

$$\delta_2 = \delta_2^e + \delta_2^t = 1 + \delta_2^t \tag{19}$$

at state ②, the bar is subjected to a decreasing load up to the load carrying capacity in compression. The subscripts refer to the states with encircled consecutive numbers in Fig. 2. If the bar is so slender that $n_E \leq 1$, it will buckle at $n = -n_E$, under which the deflection increases elastically from zero until the yield limit is reached at the bar center, according to assumption 4. This process is represented in Fig. 2 by states ② → ③ → ④. For ② → ③, $\delta = n + \delta_2^t$ with vanishing δ^g and δ^p. δ^g contributes to the relative axial displacement from state ③, caused by the lateral deflection, and δ_4^g, whose magnitude gives the length of the plateau ③ → ④ in Fig. 2(a), is determined from equation (13) in conjunction with equation (12) by setting $n_4 = -n_E$ and $|v_4| = \pi/2$, to be $\delta_4^g = -\alpha(1-n_E)^2/(4n_E)$.

In the case of $n_E \geq 1$, assumption 5 stipulates that the bar buckles at $n = -1$, and states ③ and ④ coincide. The treatment for this case is simpler, and the subsequent behavior is clear from the case $n_E < 1$. The latter case is assumed in the illustration of Fig. 2 (Particular curves used for demonstration in Figs. 2–4 are drawn for $\alpha = 1$ and $n_E = 0{\cdot}6$).

At state ④ a yield hinge is formed at the bar center, and an increasing deflection beyond this state is accompanied with the plastic rotation and plastic axial deformation at the hinge. It is seen from equation (12) that equilibrium is maintained only under decreasing compression. This process is represented in Fig. 2 by the variation ④ → ⑤, with the stress point on the yield polygon in the fourth quadrant of the stress plane Fig. 2(d). Each component of δ in equation (3) now has a nonvanishing value; δ^e, δ^g and δ^p are determined by equations (4), (13) and (18), respectively, by making use of equations (9), (12) and (14), and $\delta^t = \delta_2^t$.

If the bar ends begin to separate anywhere along the curve ④ → ⑤, such as at state ⑤, the yield hinge disappears, leaving a slope discontinuity at the bar center. During the elastic recovery ⑤ → ⑥, v decreases as in equation (16), where θ remains constant at θ_5. Increasing n is associated with increasing δ, and the latter is expressed as a function of the former by equation (3) in which δ^e is given by equation (4), δ^g by equation (15) together with equation (9), $\delta^p = \delta_5^p$ and $\delta^t = \delta_5^t = \delta_2^t$.

Elastic behavior ceases to continue at state ⑥, at which the yield limit is reached with positive n. The values n_6 and v_6 are found, as functions of the history dependent quantity θ_5 and the parameter n_E, by making use of equations (9), (12) and (16), i.e., by solving the simultaneous equations

$$n_6 = \frac{1}{1+v_6}, \qquad v_6 = \theta_5 \frac{\tanh[\pi/2\sqrt{(n_6/n_E)}]}{\pi/2\sqrt{(n_6/n_E)}}. \tag{20}$$

Plastic action again takes place along ⑥ → ⑦ with the stress point on the yield polygon in the second quadrant. θ as well as v decreases. Each component of δ in equation (3) can be

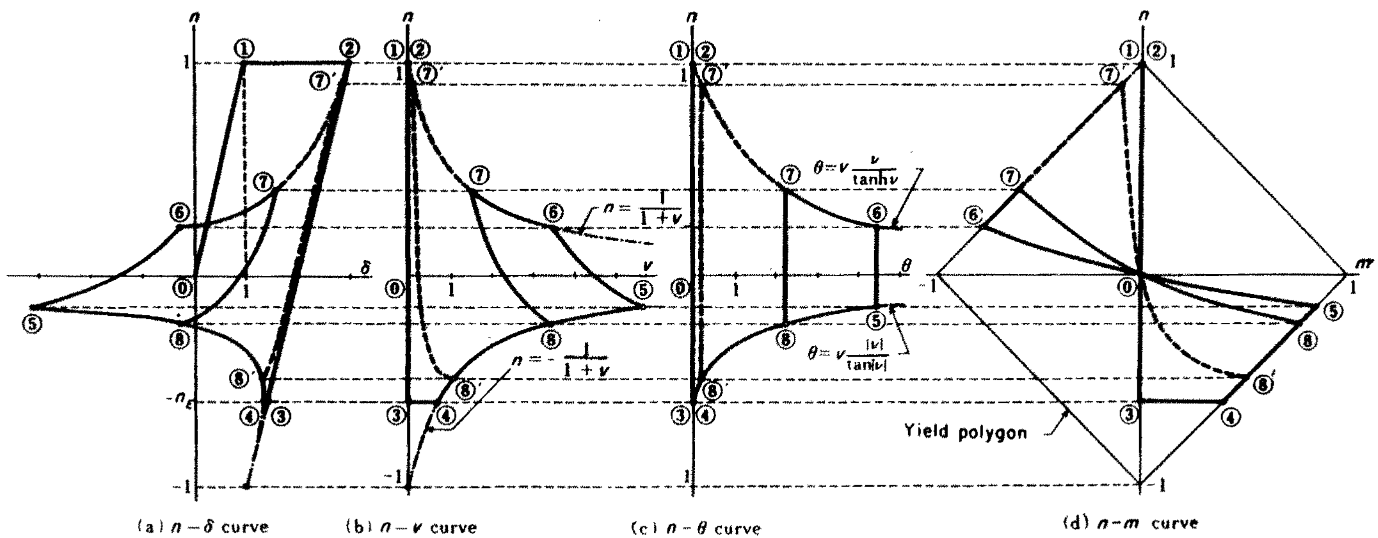

FIG. 2. Behavioral diagrams of a bar under repeated axial loading.

determined from the foregoing equations in a manner similar to ④ → ⑤, having in this case positive values of n. If the load is kept increasing and $n = 1$ is attained, then equations (12) and (14) indicate that $v = \theta = 0$, so that $\delta^g = \delta^p = 0$, and δ amounts to $1+\delta^t_2$, which is nothing but δ_2 according to equation (19). As a consequence, it is observed that the extension of the curve ⑥ → ⑦ leads to ②, as shown in dashed-and-dotted lines in Fig. 2. The bar is not straightened and does not recover its full strength, until elongated beyond $\delta = 1+\delta^t_2$. This cycle of loading and unloading, from and to state ②, has no influence whatsoever on the subsequent behavior of the bar; within the limitation of small deformation* setted in assumption 3, the bar at state ② cannot be distinguished in behavior from state ①.

Unloading from state ⑦ gives rise to elastic behavior and can be treated in a way similar to ⑤ → ⑥. The yield limit is again reached at ⑧. The ensuing plastic action is associated with $n < 0$, and δ^e, δ^g and δ^p are determined by the same equations as for ④ → ⑤; δ^t still remains constant at δ^t_2. Accordingly, it is seen that state ⑧ falls on a certain point between ④ and ⑤ in each curve of Fig. 2. The subsequent behavior is therefore clear from the foregoing descriptions. If the unloading point ⑦′ is close to ②, the traces corresponding to ⑦′ → ⑧′ tend to curve sharply near $n = -n_E$, except for n-θ curve of (c), as depicted in dashed lines in Fig. 2. The qualitative behavior, however, is not different from the case ⑦ → ⑧.

From this example it is evident that the foregoing equations are adequate for the explicit representation of the load-deformation characteristics of a bar under any history of axial loading.

4. FURTHER REMARKS

Among the components of δ in equation (3), δ^t can take any positive value, as already mentioned. δ^t is controlled by displacement constraints. δ^e is, of course, independent of the history. When plastic action takes place at the yield hinge, it is revealed from equations (9), (12), (13), (14) and (18) that δ^g and δ^p are also independent of history; they are functions only of the current value of the axial force for a given bar. Consequently, it turns out that the difference $\delta-\delta^t$ can be expressed as a function of n alone for the parameters α and n_E given, as far as plastic action occurs. This relation is shown in thick solid lines in Fig. 3. The irreversible nature of the plastic process restricts the variation in the direction shown by arrows, i.e., in the direction $\dot{n} > 0$. The variation along the lower curve is associated with increasing θ, and the upper decreasing θ. Analytically the difference in shape between the two curves results from that between trigonometric and corresponding hyperbolic functions.

Bridging between these curves is associated with elastic behavior and is reversible. $\delta-\delta^t$ depends not only on n but also on the previous history. This is because in an elastic process δ^g and δ^p depend on θ, as seen from equations (15) and (18), θ remaining constant in each elastic process. These relations are shown in thin solid lines in Fig. 3. The slope of these curves becomes larger as θ decreases, and takes the maximum and constant value unity for $\theta = 0$. Along each curve, the slope increases as n does.

With these in mind it is seen that the curves in Fig. 3 suffice for the completion of the hysteretic n–δ relation for any given history of axial loading. For instance, if the bar is subjected to cyclic displacement changes between $\delta = -\delta_A$ and $\delta = \delta_A$ ($\delta_A > 1$) with

* If the elongation attains the order of magnitude of the original bar length, the buckling strength becomes smaller than the original, to cite an instance.

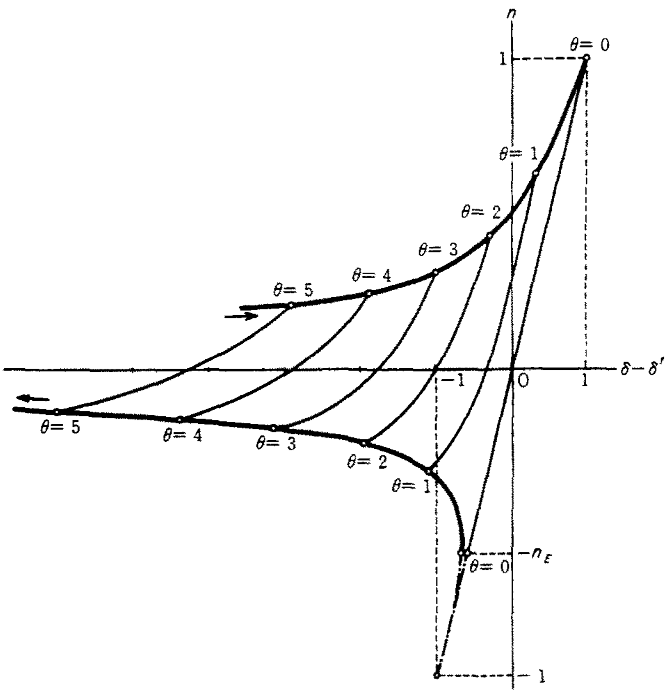

FIG. 3. n vs. $\delta - \delta'$ relation.

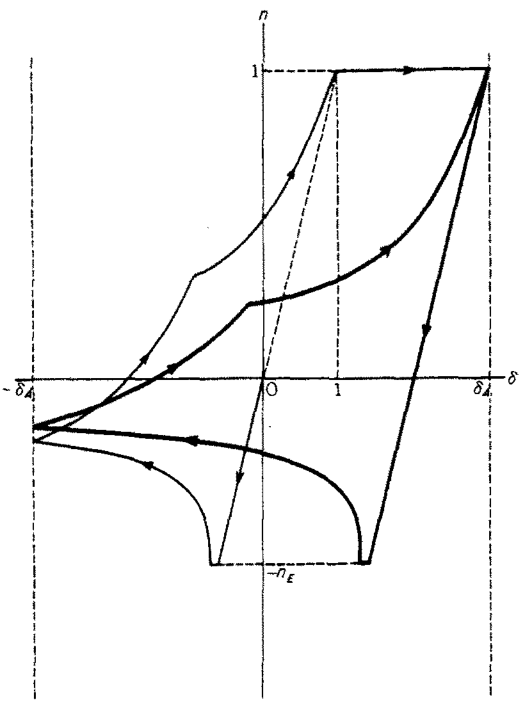

FIG. 4. Example of cyclic loading with constant displacement amplitude.

contraction first, the hysteretic n–δ relation is determined as in Fig. 4 by simple translation of curves in Fig. 3. The trace of n–δ relation falls on the final loop, the thick solid-lined curve, and becomes stable before the end of the first cycle.

5. CONCLUSIONS

The closed-form solution has been found for the hysteretic behavior of an elastic–perfectly plastic bar with moderate slenderness under repeated axial loading, based on the piecewise-linear yield condition, equation (1), for the combined action of axial force and bending moment. The solution is complete in the sense that the load-deformation characteristics can be found uniquely whenever the history of the end displacements is specified. The dimensionless relative axial displacement δ is expressed as a function of the history of the dimensionless axial load n, with the cross-sectional factor α and the dimensionless Euler buckling load n_E as parameters.

As a result of the analysis, it is observed how a bar plastically deformed due to instability recovers its strength and rigidity during the course of the subsequent elongation. The analysis also demonstrates the feature that a bar gets loosened due to plastic elongation, and the bar cannot exhibit the full strength subsequently, until it undergoes the maximum relative displacement that it has experienced. A cycle, from and to this maximum displacement, has no influence on the subsequent behavior of the bar.

It seems pertinent to conclude with critical comments on some of the basic assumptions adopted in this paper. The assumption of perfect plasticity in the use of stress resultants and the corresponding kinematical quantities neglects the stage of partial yielding in a cross-section. It is impossible therefore for the residual nature of plastic deformation to be strictly taken into consideration by the theory which has a basis on this assumption. This will cause some error in the prediction of the behavior of actual sections with a web and/or finite flange thickness under repeated loading, in addition to the error usually observed in the case of monotonic loading. To draw an instance from Fig. 2, an actual bar with elastic–perfectly plastic stress–strain relationship, after being subjected to the loading program like ① to ⑦, would not acquire full recovery by the attainment of the maximum relative displacement that the bar has previously experienced; that is, $n < 1$ at $\delta = \delta_2$. There would still remain some residual deflection at $\delta = \delta_2$ due to nonuniform residual strain distribution in a cross-section, and the load carrying capacity in the subsequent compression would be lower than that for the original state, n_E. The full strength in tension, $n = 1$, would be reached at a still larger displacement than δ_2. In order to gain access to the real behavior of an actual steel brace or a truss member, a more realistic hysteretic constitutive relation may have to be used. Nevertheless, the analysis presented in this paper has a merit in its simplicity, and will serve as a reasonable first order approximation. It is worth mentioning that for the purpose of determining the post-buckling deformation characteristics of a bar the piecewise-linear approximation of the yield condition is reasonable even for a rectangular cross-section [14]. The shape of the exemplified curves in Figs. 2 and 4 indicates qualitative agreement with the experimental results obtained from bar specimens with rectangular cross-section [1–3]. Details of the comparison between the theory presented and the experimental observations shall be discussed in a paper to follow.

For very stubby bars to which the present analysis is not accommodated because of assumption 5, the shear effect may also come into play unless the bar has a compact cross-section, or it may be too stocky to be treated as a one-dimensional continuum. If the bar is extremely slender and flexible, and forced into large deflection, assumption 3 of small deformation may not be appropriate. Such a bar, however, may well be treated as incapable of carrying compression, and can be analyzed in a very simple manner.

Mention should be made of two recent investigations, which have been brought to the author's attention since submission of the paper for possible publication: B. HIGGINBOTHAM and R. D. HANSON, *Inelastic cyclic behavior of axially-loaded members*, Summary of a talk presented at the Column Research Council Technical Session on 21 March 1972 in Chicago, and S. IGARASHI, K. INOUE, M. KIBAYASHI and M. ASANO, *Hysteretic characteristics of steel braced frames—Part* 1, *The behaviors of bracing members under cyclic axial forces*. Trans. Arch. Inst. Jap, **196**, 47 (in Japanese) (1972).

Higginbotham and Hanson utilize elliptic integrals in their analysis in order to admit large curvature. The analysis follows the notion of a yield hinge, without accounting for its plastic axial deformation. Their temporarily reported experimental results seem to agree qualitatively with the theory presented in the present paper, though indicating some discrepancies in details, e.g., in the shape of the load–displacement curve near the post-buckling range.

Based on the parabolic yield condition, the analysis of IGARASHI *et al.* resorts to step-by-step calculation. A large number of numerical results are presented in the form of charts for the case of cyclic alternate loading with constant displacement amplitude, and it is fully discussed how the load-deformation characteristics are influenced by the bar slenderness and the number of cycles of repetition.

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Абстракт—Приводится здесь упруго-идеально пластический анализ для стержня под влиянием многократной осебой нагрузки. Учитывается стержень в смысле одномерной сплошной среды, с двумя концами свободно опертыми. На основе кусочно-линейного условия текучести, рассматривается пластическое взаимодействие для совместного действия изгиба и осебой деформации. Определяется решение в замкнутом виде. Оно может описать гистерезисное поведение стержня, в смысле связей конструкции или элемента фермы, под влиянием какой-нибудь заданной истории растяжения, либо сжатия или соответствующих перемещений. Иллюстрируется примером и обсуждаются некоторые выводы аналитических результатов. Работа заключаетяс критическими замечаниями основных предположений, принятых в течении анализа.